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Quasi-2D Simulation of Liquid Metal Flow Past a Cylinder in a Duct Exposed to a Magnetic Field

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Abstract

The present study numerically investigates the fluid flow and heat transfer of a liquid metal in a rectangular duct past a circular cylinder under a strong transverse magnetic field using a spectral element method. Under these conditions, the flow is quasi-two-dimensional and the flow equations are solved over the two-dimensional domain. A parametric study is performed for Reynolds numbers $100 \leq Re \leq 3000$, Hartmann number $500 \leq Ha \leq 1200$, and blockage ratio $0.1 \leq \beta \leq 0.5$. A constant Pradntl number Pr = 0.022 is maintained. The transition of the flow from steady to unsteady flow regimes is determined as a function of Ha and β. The effect of Hartmann number and blockage ratio on the critical Reynolds number, Strouhal number, and the heat transfer from the heated wall to the fluid are investigated. The transition from steady to unsteady flow was found to increase with increasing Ha for all values of β. For blockage ratio $\beta \leq 0.2$, there was a slight change in the amount of heat transfer as Ha is increases. However, there was a significant change in the the heaat transfer at different Ha for $\beta \geq 0.2$.

Introduction

The study of liquid metal flows in ducts in the presence of a transverse magnetic field has received attention because of its applications in important technologies such as metallurgical processing, hydrodynamic generators, pumps, blood flow meters and in particular magnetic confinement fusion reactors, where the liquid metal is used as a coolant and as a breeder material [1]. In the most fusion reactor blankets the liquid metals circulate in an electrically insulated duct perpendicular to the applied magnetic field. The motion of liquid metal in a strong magnetic field induces electric currents which in turn interact with the magnetic field resulting in a Lorentz force, which has a significant effect on the velocity distribution and the turbulence characteristics, and exerts a retarding force on the flow.

The flow under a strong magnetic field is characterized by a laminar flow structure because the velocity fluctuations along the direction of the magnetic field are strongly damped. Therefore, the heat transfer in the ducts of the first wall where large amount of heating power must be removed will be dramatically decreased.

However, the two dimensional turbulence that consists of vortices with axis parallel to the magnetic field are not significantly damped [2].This turbulence could be used to enhance the heat transfer by using turbulence promoters such as a circular cylinder placed inside a duct of the blanket.

For the case without a magnetic field, the flow past a circular cylinder depends on the Reynolds number and whether the flow is confined or unconfined. In an unconfined flow domain, the flow is depends on the Reynolds number as the sole control parameter. As Reynolds number increases, the flow becomes successively unstable to two-dimensional disturbances, followed by three-dimensional instability modes [3].

However, when the circular cylinder is confined in a plane channel, the nature and the stability of the resulting flow and and flow parameters such as drag and lift coefficients, Strouhal number, and Nusselt number will be changed significantly [4]. In this configuration, the blockage ratio, which represents the ratio of the cylinder diameter to the distance between the parallel plates is an additional control parameter. Due to the additional dissipation at the confining walls, the transition from steady to unsteady flow is delayed and occurs at Reynolds number higher than that of unconfined flow.

The concept of using internal obstacles to induce vortices and enhance the heat transfer rate have been investigated experimentally [5, 6]. The heat transfer enhancement under fusion conditions in a thin wall and electrically insulated rectangular duct have been investigated experimentally by using turbulence promotion [7]. The results reveal that the heat transfer rate in the thin walled and insulated ducts was respectively improved by factors of 2 and 7 times that of the slug flow.

The aim of the present work is to study the dynamics and heat transfer characteristics of a quasi two-dimensional magnetohydrodynmic flow past a confined circular cylinder with a strong magnetic field for $50 \leq Re \leq 3000$, $0 \leq Ha \leq 1200$, and $0.1 \leq \beta \leq 0.4$. In particular, the effect of Hartmann number and blockage ratio on the structure of the flow and heat transfer will be investigated.

Mathematical formulation

The system of interest is a rectangular duct confining a circular cylinder placed at the centre of the duct parallel to the transverse direction and perpendicular to the flow direction. The duct walls and the cylinder are assumed to be electrically insulated. A homogeneous vertical magnetic field with a strength B of up to 2.1 Tesla is imposed along the cylinder axis. One of the walls oriented parallel to the magnetic field is heated to a constant wall temperature T_w whereas the other surfaces are thermally insulated. For a high Hartmann number, the magnetic Reynolds number Re_m, which represents the ratio between the induced and the applied magnetic field is very small. Thus,the induced magnetic field is negligible and the resulting magnetic field is imposed in the z-direction only. Under these conditions the flow is quasi two-dimensional and consists of a core region, where the velocity is invariant along the direction of the magnetic field, and a thin Hartmann layer at the wall perpendicular to the magnetic field. The quasi two-dimensional model has been derived by [8, 9], by averaging the flow quantities along the magnetic field direction, as shown in figure 1.

In this case the non-dimensional magnetohydrodynamic equations of continuity, momentum, and energy reduces to

$$\nabla_\perp \cdot u_\perp = 0 \, , \tag{1}$$

$$\frac{\partial u_\perp}{\partial t} + (u_\perp \cdot \nabla_\perp) u_\perp + \nabla_\perp p = \frac{1}{Re} \nabla_\perp^2 u_\perp - \frac{d}{a} \frac{Ha}{Re} u_\perp \, , \tag{2}$$

$$\frac{\partial T}{\partial t} + (u_\perp \cdot \nabla) T = \frac{1}{Pe} \nabla^2 T \, . \tag{3}$$

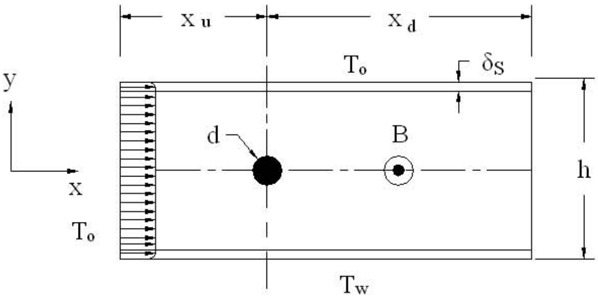

Figure 1: Schematic representation of the computational domain for the flow past a confined circular cylinder in the average plane. δ_s is the thickness of Sherclif's layer, and h and d are respectinely, the duct width and cylinder diameter.

Here u, p, T are respectively, the velocity, pressure, and temperature. The subscript $\perp$ refer to the vector quantities projected in the direction of the magnetic field, i.e. (x, y) plane. The variables in the above equations are scaled by taking d, $\rho\ U_o^2$, d/U_o and ΔT as a respective reference length, pressure, time, and temperature [10, 11].
The dimensionless parameters Re, Ha, and Pe are the Reynolds number, Hartmann number, and Peclet number, respectively. They can be written as

$$Re = \frac{U_o d}{\nu},\ Ha = aB\sqrt{\frac{\sigma}{\rho\nu}},\ Pe = RePr. \tag{4}$$

where ρ, ν, σ, B, a are respectively, the mass density, kinematic viscosity, magnetic permeability of the liquid metal, applied magnetic field, and the duct height.
No-slip boundary conditions for velocities at all solid walls are used. At the channel inlet, the transverse component of velocity is zero, and a Hartmann velocity profile for the axial velocity is applied. The temperature of the incoming stream is assumed to be T_o. At the exit, a constant reference pressure is imposed and a zero gradient velocity and temperature is imposed. The temperature at at the bottom of the channel is constant and equal to T_w, while the top wall is assumed to be at temperature T_o. A zero normal temperature gradient is imposed at the surface of the cylinder.
The flow and heat transfer characteristics in quasi two-dimensional channel shown in figure 1 for $50 \leq Re \leq 3000$, $0 \leq Ha \leq 1200$, and $0.1 \leq \beta \leq 0.4$ are investigated. A Pradntl number $Pr = 0.022$ is used, representative of eutectic alloy GaInSn.

Numerical Methodology

A spectral-element method is used to discretise the governing flow and energy equations [9]. The chosen scheme employs a Galerkin finite element method in two dimensions with high-order Lagrangian interpolants used within each element. The nodes points within each elements corresponds to the Gauss-Legendre-Lobatto (GLL) quadrature integration points producing diagonal matrices. Since the functions at the internal nodes depends on the boundary, matrix manipulation allows the internal nodes to be eliminated from the matrix subproblems of the pressure and diffusion sub-steps through static condensation. A constant reference pressure is imposed at the outlet, and a high order Neumann condition is imposed on the Dirichlet velocity boundaries to preserve the third-order time accuracy of the scheme.

In this approach, the computational domain is divided into a series of macro-elements. These elements can be refined in areas of the domain that undergo high gradients. This is known as h-refinement. The order of Lagrangian polynomial could be varied from 4 to 9 to improve the grid resolution. This is known as p-refinement. The coupled between h-refinement and p-refinement called h-p elements method.
A thorough grid resolution study was performed to ensure adequate domain sizes, and spatial and temporal resolutions to accurately resolve all features of the flow field for the Reynolds numbers, the Hartmann numbers and the blockage ratios under consideration in this study. For each blockage ratio, three families of meshes were tested. The pressure and viscous components of the drag C_{dp} and C_{dv}, and the Strouhal frequency of vortex shedding St were monitored, as they are known to be sensitive to the domain size and resolution.The upstream and downstream domain length chosen for this study for blockage ratios $\beta = 0.1$ and $\beta = 0.4$ are shown in table 1. Initially, elements with polynomial degree 7 were used for the simulations, at $Re = 3000$ and Hartmann number $Ha = 1200$, which was sufficiently large to produce periodic flows for the all meshes employed at each blockage ratio. The computation results revealed a difference of less than 1% compared with the values of St, Cd, Cd_p, and Cd_v for M_2. Therefore, the model used throughout this study will employ M_2 for all blockage ratios.
The spatial resolution study have been performed by varying the element polynomial degree between 4 and 9 within each macro-element of the mesh based on the domain length parameters from the the mesh domain study. The macro-element distribution remains unchanged throughout the spatial resolution study. As with the domain size, the flow field parameters St, Cd, Cd_p, and Cd_v are recorded. An accuracy in the order of 0.3% is desired for the selected node resolution, which is achieved with elements with polynomial degree ≥ 7.

	$\beta = 0.1$			$\beta = 0.4$		
	$\boldsymbol{M_1}$	$\boldsymbol{M_2}$	$\boldsymbol{M_3}$	$\boldsymbol{M_1}$	$\boldsymbol{M_2}$	$\boldsymbol{M_3}$
$N_{element}$	1172	1340	1484	1052	1196	1308
X_u	5	8	12	5	8	12
X_d	15	25	40	15	25	40

Table 1: Domain length parameters defining the meshes. $N_element$ is the number of elements, and X_u and X_d describe the inlet and outlet domain sizes, respectively.

Analytic velocity profiles showing Hartmann number variation in a duct without a cylinder present are plotted in figure 3. These profiles were imposed at the inlet to the computational domain. The asymptotic velocity profile is flattened for Hartmann numbers $Ha \gg 1$, and approaches the quadratic profile of Poiseuille flow as $Ha \to 0$.

The quasi-two-dimensional approximation has been validated by comparing the results of the present computations with previous laboratory experiments [6]. In that study, the critical Reynolds number (Re_c) for the transition from steady to unsteady flow was determined at $\beta = 0.1$ and a range of Ha. Figure 2 demonstrates the very close agrement (both in value and trend) between the present computations and the earlier experiments. This provides great confidence that the quasi-two-dimensional model accurately reproduces the physics of the full three-dimensional duct flow for the parameter ranges investigated in this study.

Results and discussion

The variation of the critical Reynolds number Re_c and associated critical Strouhal number St_c with Hartmann number Ha for

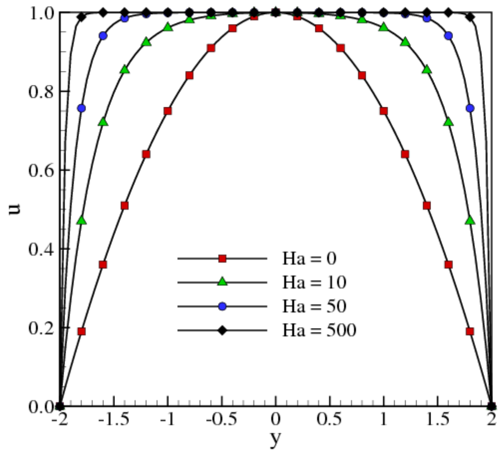

Figure 2: Base flow velocity profile at different values of Hartmann number.

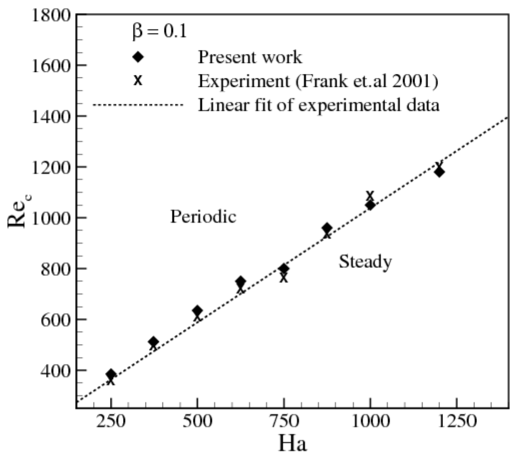

Figure 3: Comparison of critical Reynolds for the transition from steady to periodic flow with the results from other studies at different Hartmann number for a blockage ratio of 0.1.

$\beta = 0.1 - 0.5$ is shown in figure 3 and 4. For a given β, the Re_c for the transition from steady to periodic flow increases with increasing Ha. The increase in Re_c is more pronounced at high Ha and β. This is attributed to the effect of Ha and β which delay the transition from steady to periodic flow regimes, resulting in enhanced stability of the flow. The Ha number (magnetic field) shifts the exponential growth of instabilities through the linear damping action of the Hartmann boundary layer. Therefore, a higher Re is required to reach the transition to the periodic flow regime compared to the case without a magnetic field ($Ha = 0$). In addition, as the cylinder moves closer to the confined walls (higher blockage ratios), the interaction of the wall boundary layer with that of the cylinder suppress the wake instability from the cylinder.

For $\beta \leqslant 0.3$, there is a significant increase in the Re_c as Ha rises. However, for $\beta \leqslant 0.4$, Re_c increased only slightly. This is may be attributed to the fact that as the lateral walls approach the cylinder, the local acceleration of the flow near the cylinder causes it to experience a high Re flow and, therefore, makes it more unstable.

For small blockage ratio, the structure of the Karman vortex

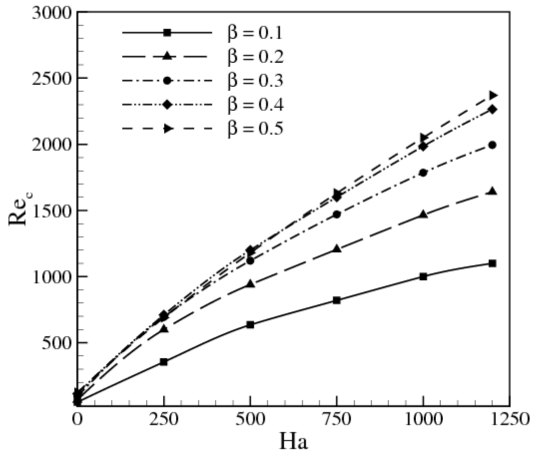

Figure 4: Variation of critical Reynolds number for the transition from steady to periodic flow with Hartmann number at blockage ratio as indicated.

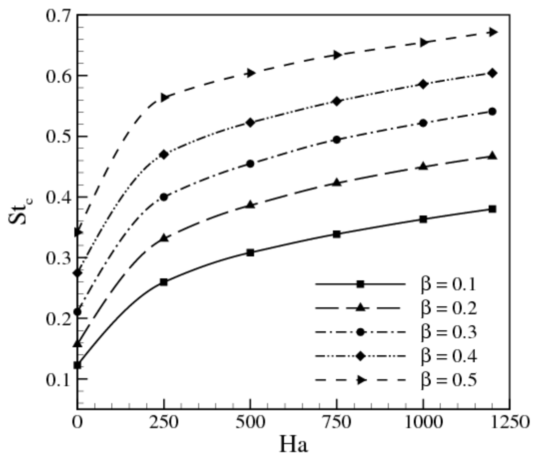

Figure 5: Variation of critical Strouhal number with Hartmann number at blockage ratio as indicated.

street consists of regular positive and negative vortices shed respectively from the bottom and the top shear layers. At high Hartmann number, $Ha = 1200$, the strength of the vortex street decreases and the released vortices are compacted close together, and they diffuse rapidly further downstream. For $\beta = 0.3$, at Hartmann number of 500, boundary layer entrainment from the walls (the Shercliff layers) occurs downstream of the cylinder. The structure of the vortex street is regular, closely-spaced compact vortices shed from the bottom and the top of the cylinder. The boundary layer detachment from the walls increased gradually as the blockage ratio increases to 0.4 and 0.5 at $Ha = 500$. It sheds and begin to interact with the Karman street, which creates an obstacle that impedes its motion. However, at $Ha = 1200$, the vortex shedding is completely suppressed, as shown in figure 6.

Figure 7 presents the distribution of instantaneous temperature contours at $Re = 2000$ for blockage ratios of $\beta = 0.3$ and 0.4,

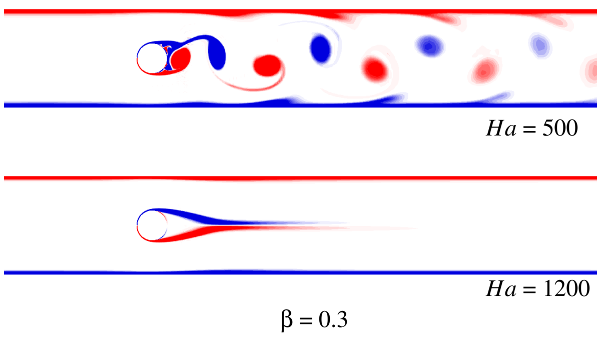

Figure 6: Vorticity contour plots for $Re = 2000$ and β of 0.3 at low and high Hartmann number. 20 contour level are displayed between $-2 \leq \omega \leq 2$, with red and blue contours representing negative and positive vorticity.

at Hartmann numbers $Ha = 500$ and 1200 . For all β, when $Ha = 500$, the temperature field is time-dependent because the flow is unsteady at this Hartmann number. As Ha is increased to 1200, the unsteadiness in the flow is weak for $\beta = 0.2$. However, the flow becomes steady as β increases to 0.3. Therefore, the thickness of the thermal boundary layer is larger than that of duct side layer (Shercliff layer) and as a result the thermal boundary layer is not significantly affected by the Shercliff boundary layer.

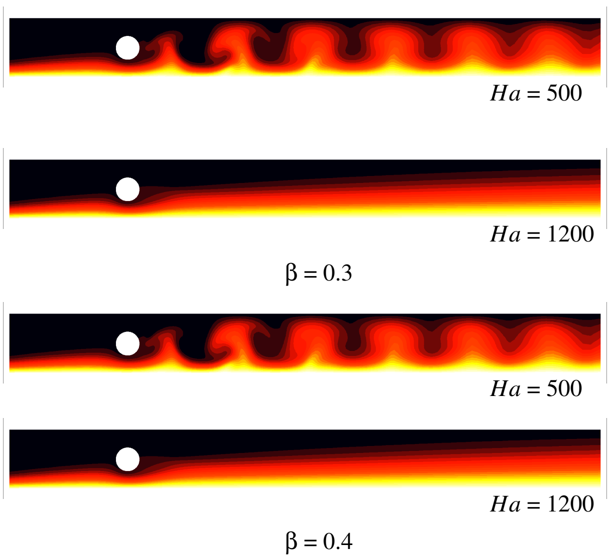

Figure 7: Instantaneous dimensionless temperature contours at $Re = 2000$ for $\beta = 0.3$ and $\beta = 0.4$ at low and high Hartmann number. 20 contour level are displayed between $0.05 \leq T \leq 0.95$, with red and black contours representing warmer and colder temperatures respectively.

Conclusions

The present study numerically investigates the characteristics of liquid metal flow and heat transfer past a circular cylinder in a rectangular duct under a strong axial magnetic field using the spectral-element method. Under these conditions, the flow is quasi-two-dimensional and the modified Navier-Stokes equations are solved in a two-dimensional domain.
The critical Reynolds number at which the transitions takes place from steady to unsteady flow was found to increase with increasing Ha for all values of β.
The heat transfer was higher for higher blockage ($\beta = 0.4$) at low Hartmann number ($Ha = 500$). However, for $\beta \geq 0.4$, it decreased with increasing Ha. For $\beta \geq 0.2$, a significant increase in the heat transfer was found with increasing β at $Ha = 500$. However, at $Ha = 1200$, no variation with β was observed.

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