2011 International Conference on Mechanical, Industrial, and Manufacturing Engineering
Lecture Notes in Information Technology Vols. 1-2

Optimal Transient Disturbances Preceding Vortex Shedding in Magneto-Hydrodynamic Flow past a Circular Cylinder in a Duct

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***Abstract* – Flow of liquid metal in an electrically insulated rectangular duct past a circular cylinder under a strong axial magnetic field is investigated in the subcritical regime, below the onset of von Kármán vortex shedding. In this configuration, the flow is quasi-two-dimensional and can be solved over a two-dimensional domain. A transient energy growth analysis of optimal linear perturbations is carried out. Parameters are considered for Reynolds numbers $50 \leq Re \leq 2500$, Hartmann numbers $500 \leq Ha \leq 1200$, and blockage ratios $0.1 \leq \beta \leq 0.4$. Transient growth is determined as a function of the evolution time of disturbances. For all β, the energy amplification of the disturbances was found to decrease significantly with increasing *Ha* and the peak growths shift towards smaller times. The nature of the disturbance does not vary with the blockage ratios.**

***Index Terms* – transient growth, magnetohydrodynamic, quasi-two-dimensional, circular cylinder.**

I. Introduction

The study of flows of electrically conducting fluids in ducts in the presence of a transverse magnetic field is important because of its practical applications in magnetohydrodynamic generators, pump, and metallurgical processing. The primary application motivating this study is magnetic confinement fusion reactors, where liquid metal is used as a coolant and as a breeder material [1]. In most fusion reactor blankets, the liquid metal circulates in an electrically insulated duct perpendicular to the applied magnetic field. The motion of liquid metal in a strong magnetic field induces electric currents, which in turn interact with the magnetic field resulting in a Lorentz force. This has a significant effect on the velocity distribution and turbulence characteristics, and exerts a retarding force on the flow.

The flow under a strong magnetic field is characterized by a laminar structure because the velocity fluctuations in the direction of the magnetic field are strongly damped. Therefore, the heat transfer in the ducts of the blanket where a large amount of heat must be removed is dramatically decreased [2]. However, two-dimensional turbulence that consists of vortices with axes parallel to the magnetic field are not significantly damped [3]. This turbulence could be used to enhance the heat and mass transfer by using turbulence promoters such as a circular cylinder placed inside the duct of a blanket.

The concept of using internal obstacles to induce vortices and enhance the heat transfer rate has been investigated experimentally by [4, 5] and numerically by [6]. The results reveal that the heat transfer rate under a strong axial magnetic field in insulated ducts was improved by a factor of more than 2 times that of laminar flow.

Stable flow may be sensitive to transient growth of disturbances for some time before decaying to zero [7]. In purely hydrodynamic parallel shear flows (*Ha* = 0), transient growth has been demonstrated for the plane channel [8], pipe [9], rectangular duct [10], and abrupt geometrical expansion flows [11, 12]. This growth can be attributed to the non-normality of the eigenmodes associated with many shear flows [7, 11]. For the cylinder wake without magnetic field (*Ha* = 0), the adjoint and direct eigenmodes in the region of primary instability has been investigated numerically by [13, 14] to understand the sensitivity of the flow. More recently, the transient response of the subcritical and supercritical flow has been investigated by [15] and [12], respectively. The transient growth in supercritical and subcritical flow of the circular cylinder wake in an open flow has been studied by [16].

The effect of an applied magnetic field on the transient growth for the case of steady Hartmann flow (channel flow of an electrically conducting fluid in presence of uniform magnetic field) has been analyzed by [17-19]. The optimal modes were found to have the form of streamwise rolls confined to the Hartmann layers. In addition it was found that energy gains of the optimal perturbation are proportional to $(Re/Ha)^2$, and the critical Reynolds number was much higher than for Poiseuille flow.

More recently, [20] analyzed the optimal linear growth of perturbations in a rectangular duct with different aspect ratio subjected to a uniform transverse magnetic field. The disturbances of optimal growth are confined to the Shercliff layers. The optimal perturbations are significantly damped by the magnetic field irrespective of the duct aspect ratio. They conclude that the Hartmann boundary layers perpendicular to the magnetic field is not contribute to the transient growth.

The aim of this paper is to quantify and analyze the transient growth of infinitesimal perturbations in a quasi-two-dimensional MHD flow past a confined circular cylinder exposed to a strong magnetic field in the subcritical regime prior to the onset of vortex shedding. This research may lead to improved mechanisms for enhancing heat transfer in strongly magneto-hydrodynamically damped duct flows.

Parameter ranges to be considered are $50 \leq Re \leq 2500$, $500 \leq Ha \leq 1200$, and $0.1 \leq \beta \leq 0.4$. In particular, the effect of Hartmann number and blockage ratio on the transient growth will be investigated.

II. Mathematical Formulation

The system under investigation is a rectangular duct confining a circular cylinder placed at the center of the duct parallel to the transverse direction and perpendicular to the flow direction. The duct walls and the object are assumed to be electrically insulated. A homogeneous vertical magnetic field with a strength *Bo* of up to 2 Tesla is imposed along the cylinder axis. For a high Hartmann number, the magnetic Reynolds number Re_m, which represents the ratio between the induced and the applied magnetic field, is very small that the induced magnetic field is negligible and the resulting magnetic field is imposed in the *z*-direction only. Under these conditions the flow is quasi two-dimensional and consists of a core region, where the velocity is invariant along the direction of the magnetic field, and thin Hartmann layer at the wall perpendicular to the magnetic field. The quasi two-dimensional model has been derived by [21, 22], by integrating the flow quantities along the magnetic field direction as shown in Fig. 1.

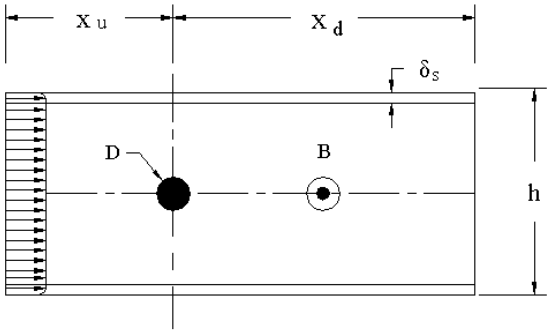

Fig. 1: Schematic representation of the computational domain for the flow past a confined circular cylinder in the average plane

In this case the non-dimensional magnetohydrodynamic equations of continuity and momentum [22, 23] are

$$\nabla_\perp \cdot \mathbf{u}_\perp = 0 \tag{1}$$

$$\frac{\partial \mathbf{u}_\perp}{\partial t} + (\mathbf{u}_\perp \cdot \nabla_\perp)\mathbf{u}_\perp + \nabla p_\perp = \frac{1}{\mathrm{Re}}\nabla_\perp^2 \mathbf{u}_\perp + \frac{d}{a}\frac{Ha}{\mathrm{Re}}\mathbf{u}_\perp, \tag{2}$$

where the variables are scaled by taking d, ρU_0^2, and d/U_0 as a respective length, pressure, and time. The dimensionless parameters *Re* and *Ha* are the Reynolds number and Hartmann number, respectively. They are defined as

$$Re = \frac{U_0 d}{\nu} \tag{3}$$

$$Ha = \sqrt{\frac{\sigma}{\rho \nu}} \tag{4}$$

The linearised Navier-Stokes equations are derived by substituting velocity and pressure fields decomposed into a two-dimensional base flow and an infinitesimal fluctuating components $\mathbf{u}'(x,y,t)$ and $p'(x,y,t)$. The linearised expansion is based on a steady two-dimensional base flow.

The maximum energy growth of initial perturbations for a given time period can be expressed as an eignvalue problem in which the perturbation can be expressed in terms of a set of optimal modes which grow at different amplitudes over the chosen time interval. The relative energy amplification (*G*) of an optimal mode is written as

$$G(\tau) = \frac{E(t=\tau)}{E(t=0)}, \tag{5}$$

where $E(t)$ is the energy of the disturbance at time *t*. $G(\tau)$ is the leading eigenvalue obtained after integrating a disturbance field forward in time using the linearized Navier-Stokes equations, and backwards in time using the adjoint linearlized Navier-Stokes equations. For the linearized and adjoint equations, it has been determined that the Hartmann friction term (the last term in equation 2) appears as $+\frac{d}{a}\frac{Ha}{\mathrm{Re}}\mathbf{u}'_\perp$ and $+\frac{d}{a}\frac{Ha}{\mathrm{Re}}\mathbf{u}^*_\perp$, respectively, where $\mathbf{u}^*_\perp$ denotes the adjoint disturbance velocity field. In all other respects, the direct transient growth technique applied here is identical to that described in [11], and the linearized eigenmode solver has been validated in [23, 24].

III. Numerical Methodology

A spectral-element method is used to discretise the governing flow equations [22]. The chosen scheme employs a Galerkin finite element method in two dimensions with high order Lagrangian interpolants used within each element. The nodes points within each element correspond to the Gauss-Legendre-Labatto quadrature points.

A no-slip boundary conditions for velocity is imposed on all solid walls. At the channel inlet, a normal component of velocity is assumed to be zero, and a Hartmann velocity profile for the axial velocity is applied. At the exit, a constant reference pressure is imposed and a zero streamwise gradient of velocity is weakly imposed through the Galerkin treatment of the diffusion term of the momentum equation. A constant reference pressure is imposed at the outlet, and a high order Neumann condition for the pressure gradient is imposed on the Dirichlet velocity boundaries to preserve the third-order time accuracy of the scheme [25].

The computational domain is divided into a grid of macro-elements. Elements are concentrated in areas of the domain that undergo high gradients. A grid resolution study determined the domain size, the number of mesh elements, and the required number of nodes per element to resolve the flow features to within 1%. The meshes typically comprise 1052 to 1484 macro elements with 49 (7×7) nodes per element. The inlet length and outlet length were $8D$ and $25D$, respectively.

IV. RESULTS AND DISCUSSION

The base flow

The base flow is characterized by a steady recirculation region of a pair of symmetric counter-rotating vortices on either side of the wake centreline. In Fig. 2, the effect of Hartmann number and blockage ratio on the flow is presented. It can be seen that an increase in the Hartmann number acts on the wake by decreasing the length of the recirculation bubble. This is due to the domination by the Lorentz force which produces a convective motion in a direction opposite the flow, resulting in the decrease of the wake length. For $\beta = 0.1$ the recirculation bubble does not completely vanish up to Ha = 1200, but for $\beta = 0.4$, though, it is suppressed completely at Ha = 1000 (not shown).

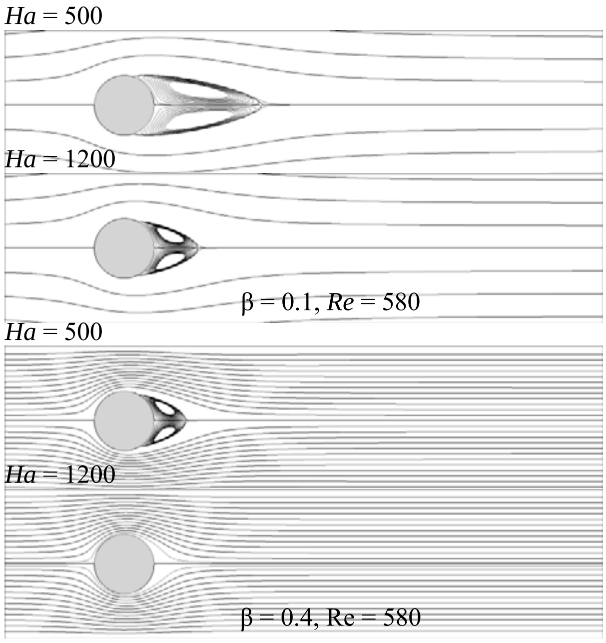

Fig. 2: Streamlines of the steady base flow at Re = 580 for blockage ratio and different Hartmann numbers as shown. Flow is left to right.

Transient growth analysis

Fig. 3 show the base-10 logarithm of the transient energy growth G of optimal disturbances as a function of evolution time τ for the steady base flow for blockage ratio $\beta = 0.1$ at different Hartmann numbers Ha. The initial observation on these data is, though the chosen Re for the analysis is well below the critical Reynolds number Re_c at the lowest Hartmann number $Ha = 500$, there exist perturbations which grow in energy by a factor of thousands. For example, for $\beta = 0.1$, the energy grows by a factor of 3.5×10^3 and 1.3×10^3 at $Ha = 500$ and $Ha = 1200$, respectively. For $\beta = 0.4$ (not shown), there is a growth of energy by a factor of 4.55×10^3 and 1.47×10^3, respectively. For all β, it is found that increasing Ha leads to a significant reduction of the energy amplification of the disturbances and to shift of the peak growth towards smaller times. In fact, the interpretation of such suppression is the reduction of perturbation kinetic energy by the Hartmann damping. Remarkably, the global maxima of energy vary significantly with the blockage ratio.

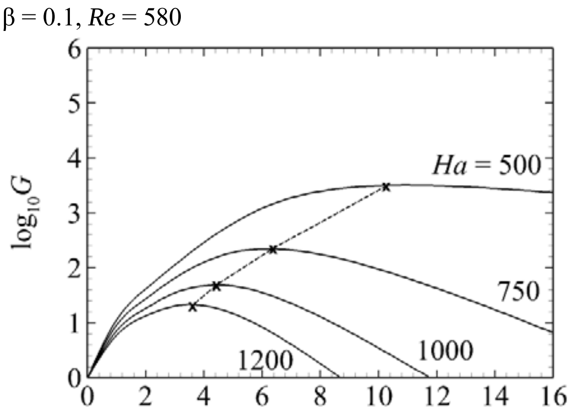

Fig. 3: Plots of $\log_{10}G$ against τ at $\beta = 0.1$, $Re = 580$, and Hartmann numbers as shown. The dashed curve shows the locus of maximum growth as a function of τ.

Fig. 4 plots the streamwise vorticity field of the optimal initial perturbations for the blockages $\beta = 0.4$ at low and high Hartmann numbers, i.e., $Ha = 500$ and $Ha = 1200$, respectively. In each, the perturbation maximum is located in the region of the boundary layer separation around the cylinder near the wake. The perturbation travels along the separating region as long as possible, hence providing the optimal growth. Remarkably, the nature of the disturbance does not vary with the blockage ratios. Fig. 5 (a, c) shows the streamwise vorticity of initial perturbation for $\beta = 0.4$, $Re = 1160$ and $Ha = 500$ and $Ha = 1200$, for which $\tau_{max} = 7$ and 2, respectively. It is clear to see that by increasing Hartmann number, the optimal perturbations are more concentrated around the boundary layer separation. In Fig. 4 (b, d), the evolution from this initial disturbance is plotted. The flow structures are a series of counter-rotating spanwise rollers, which are similar to structures seen in transient growth analysis of a cylinder in an open flow [13, 14]. Over time, vorticity detaches from the aft of the cylinder and travels downstream along the wake centreline, and a region of opposite sign voricity forms at the aft of the cylinder. In addition, boundary layer detachment from the duct side wall occurs downstream of the cylinder. Vorticity is drawn into the channel and interacts with that detached from the rear of the cylinder. The boundary layer detachment from the walls was observed to change significantly as β is further increased.

(a)

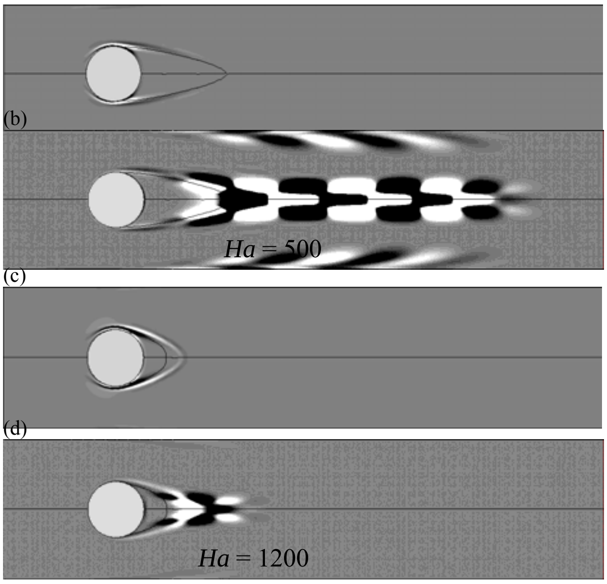

Fig. 4: Plots of stream-wise vorticity for β = 0.4 (a, c) optimal initial perturbation at *Re* = 1160 at *Ha* = 500 and 1200, respectively. (b, d) the corresponding linear growth evolved at τ_{max} for the same Reynolds numbers and Hartmann numbers of (a, c). The streamlines of the stable base flow is overlaid in each case

V. CONCLUSIONS

An investigation has been carried out into the transient growth of optimal linear perturbations of a liquid metal magneto-hydrodynamic flow past a confined cylinder in a duct under a strong magnetic field in the subcritical regime prior the onset of oscillations. Under these conditions, the flow is quasi-two-dimensional.

For all blockage ratios, very significant transient energy growth was found in this regime, which suggests a potential for the design of actuation mechanisms to promote vortex shedding and thus enhance heat transfer in these ducts. The energy amplification of the disturbances was found to decrease significantly with increasing Hartmann number, where the growth peaks at shorter time intervals. The nature of the disturbance does not vary with blockage ratio.

ACKNOWLEDGMENT

This research was supported by a Monash University Faculty of Engineering Small Grant. Computations were supported by Monash the e-Research Centre and ITS-Research Support Services through the use of Monash Sun Grid cluster, and the NCI National Facility in Canberra, Australia. NCI is supported by the Australian Commonwealth Government. W.K.H. is supported by the Ministry of Higher Education and scientific Research from the Iraqi Government.

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