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SHEAR LAYER REATTACHMENT ON A SQUARE CYLINDER WITH INCIDENCE ANGLE VARIATION

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ABSTRACT

Flow over a square cylinder at an angle of incidence (α) is computed at Reynolds numbers up to $Re = 250$ to investigate the reattachment of the shear layer on the upper surface, which is believed to play a role in the dynamics and stability of these flows. The occurrence of reattachment is correlated with the periodic lift coefficient, and the sensitivity of reattachment is examined through the employment of a boundary-layer flow control mechanism applied through the top surface of the cylinder. At $\alpha = 0°$, reattachment at the upper rear surface occurs close to the point of maximum lift force, while the duration of reattachment increases as *Re* is decreased to the critical Reynolds number for three-dimensional flow transition predicted in previous studies. Increasing the angle of incidence to 7.5° and 15° produces a weaker reattachment occurring close to the point of minimum lift and a converse relationship between duration and critical *Re*. This may be related to the correlation between incidence angle and the instability modes A and C identified in earlier studies for three-dimensional flow transition. A stability analysis will be conducted to test this hypothesis.

NOMENCLATURE

α	incidence angle
Re	Reynolds number
$\mathbf{u}$	velocity field
p	pressure field
d	side length
h	projected frontal height
U	freestream velocity
v	kinematic viscosity
St	Strouhal number
f	vortex shedding frequency
t	time as an independent flow variable
T	period of wake shedding cycle (dimensionless)
C_L	lift coefficient
C_Q	flow control coefficient
$\overline{V}_{\max}$	average velocity of profile

INTRODUCTION

Flow around a square cross-section has been studied extensively by van Oudheusden *et al.* (2007), Luo *et al.* (2003) and Sheard *et al.* (2009). This problem is highly relevant to engineering applications including flow around buildings, towers and bridge decks (van Oudheusden *et al.* 2007). In addition, bluff body aerodynamics has pertinence to jet propulsion applications such as the mixing of gases and fuel in combustion chambers, whose effectiveness is subject to the properties of the oncoming flow and the shape of the mixing apparatus.

Shear layer attachment affects the flow around a body by causing pressure losses near the body surface, close to a no-slip wall. For square cylinders, shear layer attachment can determine flow efficiency based on the inclination of the cylinder (α), the flow speed, characterized by *Re* and the resulting aerodynamics forces (Luo *et al.* 2003). The inclination of the cylinder can induce asymmetricity in the flow at certain values of α, thus influencing the downstream wake formation, the period of the vortex shedding cycle and the tendency of the wake to exhibit unstable behaviour (Luo *et al.* 2003; Sheard *et al.* 2009).

Luo *et al.* (2003) computed the flow for a square cylinder with incidence at $Re = 1000$ and $Re = 250$ to find that the higher *Re* case exhibits reattachment cycles of the upper shear layer varying between strong and weak within an alternate vortex shedding cycle. No such alternating nature was found in the lower *Re* case, which also demonstrated an absence of reattachment for lower incidence angles ($\alpha < 6°$), hence demonstrating a more stable wake. However, further investigation is required to determine the dependence of the upper shear layer attachment on the flow speed and inclination higher than $\alpha = 6°$ (Luo *et al*. 2003) at low Reynolds numbers.

Transition from two-dimensional flow to three-dimensional flow is critical in determining the stability of the cylinder. This transition was fully understood until only recently in (Sheard *et al.* 2009), as a result, the relation between shear layer attachment and regular instability mode A and sub-harmonic mode C as a function of incidence angle (Sheard *et al.* 2009) is not apparent and requires a stability analysis.

From Luo *et al.* (2003), cases with no reattachment experience a more stable wake; it is desired to control the stability of the square cylinder by altering the reattachment cycle using a suction or blowing mechanism to manipulate the flow.

Shear layer attachment can be induced by a flow control mechanism, where velocity can be imposed through the cylinder surface to counteract the effects of asymmetricity about the wake centreline and eliminate the dependence of shear layer separation and stability of the cylinder on the orientation of oncoming flow. Flow control and shear layer reattachment has not been explored in this context to date. This technique can be applied to aerospace structures such as wings and fuselages to maintain shear layer attachment, thus preventing stall and enhancing stability.

MODEL DESCRIPTION

Shear Layer Attachment without Flow Control

The model consists of a cylinder with a square cross-section with side length d, inclined at an angle of incidence, α, with oncoming freestream velocity U. Figure 1 shows the project frontal height h as a function of α so that the Reynolds number can be defined as

$$Re = \frac{Uh}{\nu}, \quad (1)$$

where $h = d(\sin(\alpha)+\cos(\alpha))$ and ν is the kinematic viscosity of the fluid. The frequency of vortex shedding (f) in the cylinder wake is defined by the Strouhal number as

$$St = \frac{fh}{U}, \quad (2)$$

where f can be found from the period of the shedding cycle using $f = 1/\mathrm{T}$.

Shear Layer Attachment with Flow Control

The flow control mechanism implemented imposes a velocity with a parabolic profile on the upper surface of the inclined cylinder as shown in Figure 1. The maximum velocity of the profile is characterized by the flow control coefficient C_Q and the average velocity of the profile as

$$C_Q = \frac{\overline{V_{\max}}}{U}, \quad (3)$$

where $\overline{V_{\max}}$ is the average velocity of the profile, in this case, two-thirds of the maximum velocity for a parabolic velocity profile, obtained from basic integration.

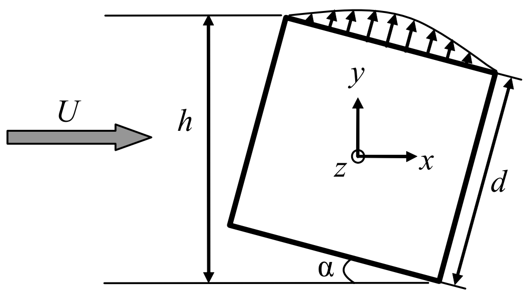

Figure 1: Schematic diagram of geometry with flow control implemented as parabolic velocity profile on top surface.

Numerical Method

The incompressible Navier-Stokes equations are discretized using a spectral-element solver in space and a third-order backwards-multistep discretization in time, previously employed and validated in Sheard *et al.* (2009),

$$\frac{\partial \boldsymbol{u}}{\partial t} = -(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - \nabla p + Re^{-1}\nabla^2\boldsymbol{u}, \quad (4)$$

$$\nabla\cdot\boldsymbol{u} = 0, \quad (5)$$

where $\boldsymbol{u}(\boldsymbol{x},\boldsymbol{y},t)$ is the two-dimensional velocity field and $p(\boldsymbol{x},\boldsymbol{y},t)$ is the pressure field. Equations (4) and (5) have been non-dimensionalized using a reference velocity and reference length, thus characterizing the flow by the non-dimensional parameter, Re.

The two-dimensional mesh is constructed of nodal quadrilateral spectral elements and identical boundary conditions are applied to the flow domain, following Sheard *et al*. (2009), with the exception of the cases where flow control requires an additional boundary condition on the top surface of the cylinder. The flow is periodic in the spanwise direction due to the utilization of a Fourier series discretization of the flow variables (Sheard *et al.* 2009).

The ongoing stability analysis of the two-dimensional flow solution applies the Navier-Stokes equations and three-dimensional perturbation fields that are evolved using linearized Navier-Stokes equations, in accordance with Sheard *et al.* (2009). The stability of the perturbations is characterized as a function of Re and m, the perturbation wavenumber. Similar to the base flow, the perturbation fields are also sinusoidal in the spanwise direction and couple only with the base flow, not other perturbations (Sheard *et al*. 2009).

RESULTS

Shear Layer Attachment without Flow Control

Figure 2 shows the vorticity of the periodic flow in the wake of the square cylinder visualized over a spanwise domain of $25d$, in agreement with Sheard *et al.* (2009), for the zero-degree incidence angle case with $Re = 200$. The flow in the wake of the cylinder for $Re = 164$ is similar to Figure 2, hence is not shown here. A similar periodic solution is seen for $\alpha = 7.5°$ and $15°$ in Figures 3 and 4, with the effect of the asymmetric incidence on the spanwise wake inclination more pronounced for the case with $\alpha = 15°$ at $Re = 245$.

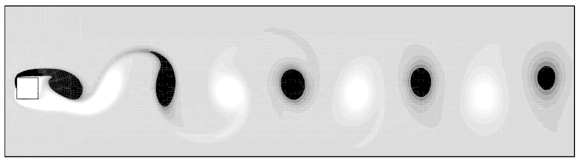

Figure 2: Two-dimensional flow in wake of square cylinder with $\alpha = 0°$, $Re = 200$.

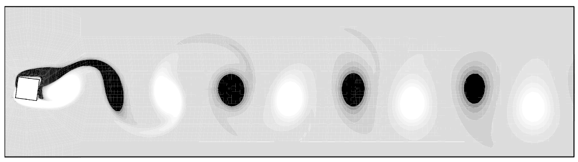

Figure 3: Two-dimensional flow in wake of square cylinder with $\alpha = 7.5°$, $Re = 224$.

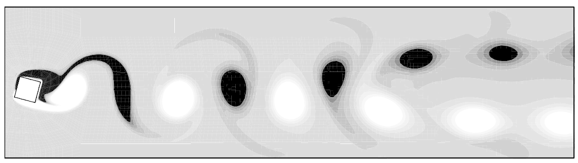

Figure 4: Two-dimensional flow in wake of square cylinder with $\alpha = 15°$, $Re = 245$.

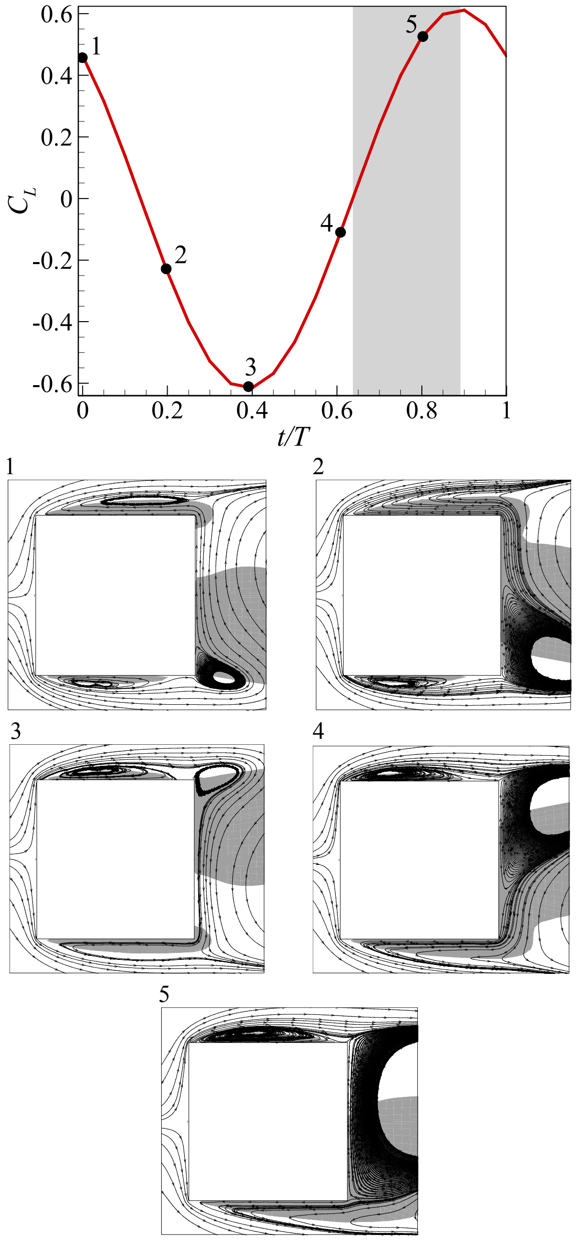

Figure 5: C_L versus time during a single period (T = 6.5762) for $\alpha = 0°$, where the shaded grey area indicates the occurrence of shear layer reattachment and corresponding streamline plots for $Re = 200$, where the grey areas indicate a negative velocity in the x-direction, signifying flow recirculation. Flow is from left to right.

The lift force coefficient (C_L) for the cylinder surface is found to vary sinusoidally with respect to time for all angles of incidence, in agreement with Luo *et al.* (2003). Figures 5 and 6 show the shear layer separation and reattachment at different points throughout a single period of the shedding cycle for $\alpha = 0°$. The streamlines show the recirculation of the flow, where the streamlines diverging away from the rear corner of the top surface indicate separation of the shear layer. Reattachment occurs as the lift force coefficient increases to its maximum value, represented by the shaded grey area in the C_L versus t/T plot in Figure 5.

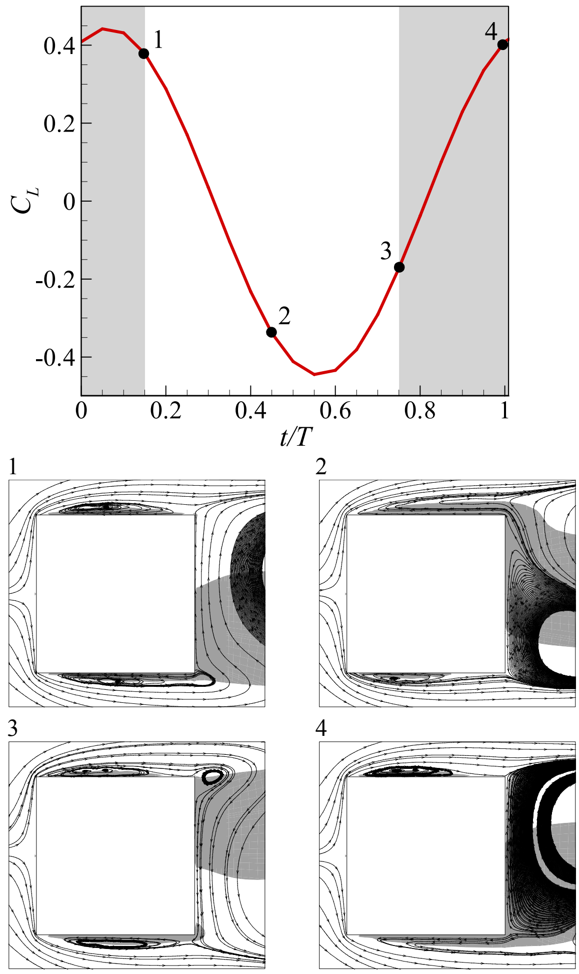

Figure 6: C_L versus time during a single period (T = 6.3956) for $\alpha = 0°$ and corresponding streamline plots for $Re = 164$. Additional plot details are identical to Figure 5.

It is also evident from Figure 6 that the duration of reattachment increases as Re decreases to the critical value of 164 (Sheard *et al.* 2009) for the case with a zero-degree incidence angle.

From Figure 7, the extent of reattachment is fairly weak compared to Figures 5 and 6, as evident from the streamline plot at the third point. Luo *et al.* (2003) observed from experimental results that the upper shear layer reattaches at $\alpha = 6°$, when $Re = 250$. Hence, it is agreeable for $\alpha = 7.5°$ to see reattachment of the upper shear layer at a similar Re. Contrary to the results from the zero-degree case, reattachment occurs near the minimum lift force coefficient, as seen from Figures 7 and 8. The duration of reattachment decreases as Re is decreased from Figure 6 to the critical Re, differing from the previous case with a zero-degree incidence angle.

Figure 9 shows that reattachment at 15° incidence is stronger than seen at 7.5°and occurs at roughly the same point in time during the shedding cycle period. The duration of reattachment follows a similar trend as the 7.5° case, confirmed by Figure 10 at $Re = 168$.

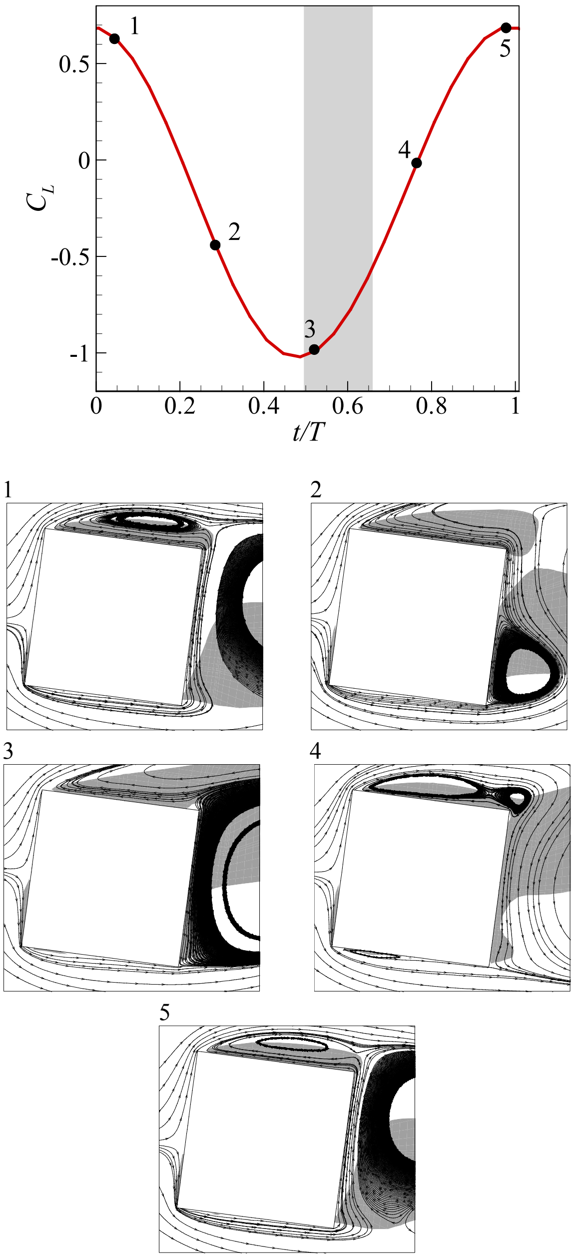

Figure 7: C_L versus time during a single period (T = 6.3510) for α = 7.5° and corresponding streamline plots for *Re* = 224. Additional plot details are identical to Figure 5.

The stronger level of reattachment at α = 15° at its critical *Re* as per Sheard *et al*. (2009) indicates that there is an association to the instability mode developed when transitioning from two-dimensional to three-dimensional flow, a characteristic dependent on the incidence angle. Sheard *et al*. (2009) states that instability mode A is dominant for angles of incidence between 0° and approximately 12°, while subharmonic mode C is the principal mode for angles between 12° and 26°. Consequently, the subharmonic instability mode can be said to induce a stronger reattachment of the upper shear layer, however this is yet to be confirmed with a stability analysis.

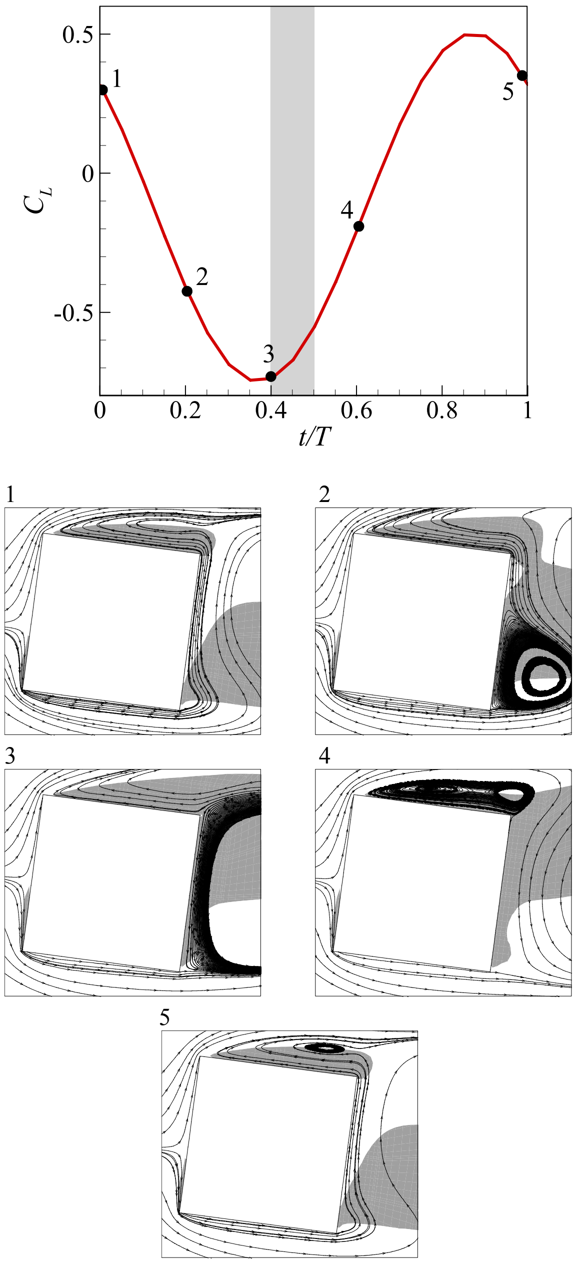

Figure 8: C_L versus time during a single period (T = 6.4676) for α = 7.5° and corresponding streamline plots for *Re* = 176. Additional plot details are identical to Figure 5.

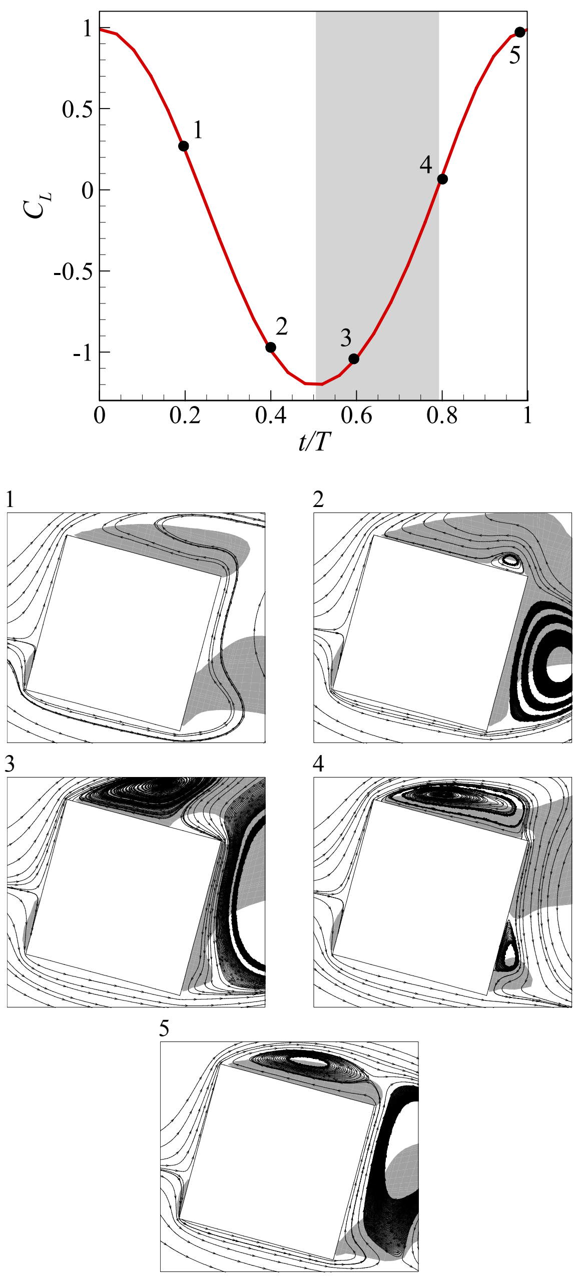

Figure 9: C_L versus time during a single period (T = 6.3528) for α = 15° and corresponding streamline plots for *Re*=245. Additional plot details are identical to Figure 5.

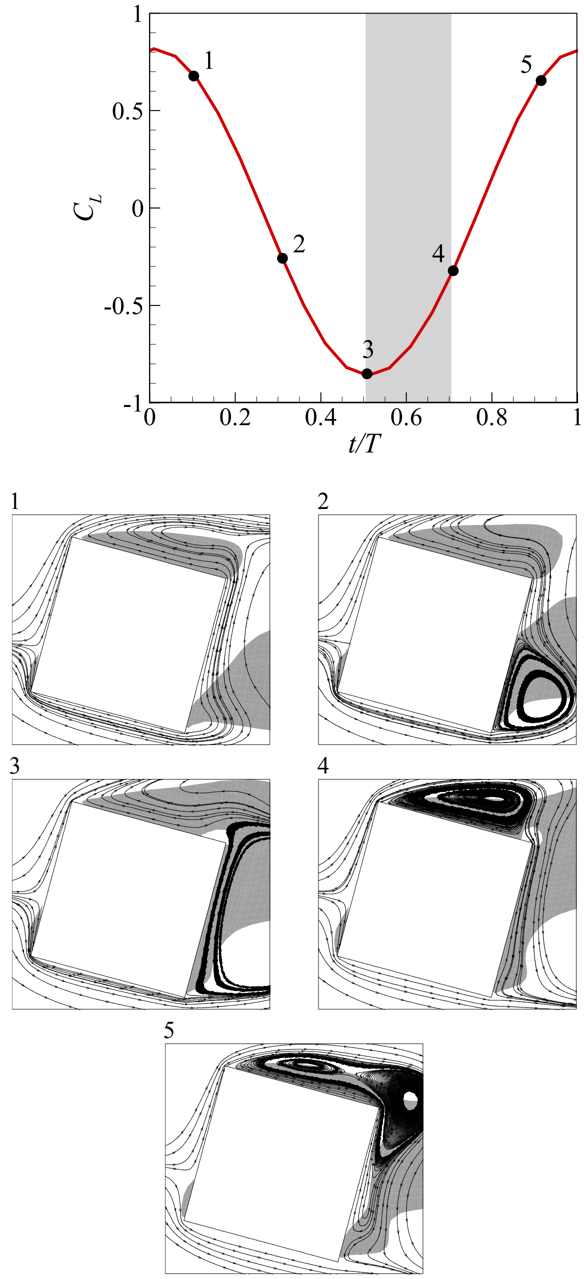

Figure 10: C_L versus time during a single period (T = 6.5965) for α = 15° and corresponding streamline plots for *Re* =168. Additional plot details are identical to Figure 5.

Shear Layer Attachment with Flow Control

It is found that the shear layer on the upper cylinder surface is highly sensitive to the flow control velocity imposed along this surface. Figure 11 shows the streamline plot for C_Q = -0.0333 and α = 7.5° at *Re* = 176, demonstrating that the region of recirculation on the top surface is eliminated, removing any reattachment or separation of the shear layer. A similar result is obtained for a positive flow control coefficient, the sole difference being that the flow is directed away from the upper cylinder surface.

The Strouhal number is monitored for various values of C_Q to determine the effect of the imposed velocity on the wake shedding frequency. A non-linear trend is established from Figure 12, with a more pronounced increase in the Strouhal number as C_Q is increased to a larger negative value. Since the flow control mechanism has not been previously investigated to this extent, this trend is not comparable to previous studies.

In addition, the average lift coefficient on the cylinder surface is plotted against C_Q in Figure 13 to observe the effect of the force exerted on the cylinder by the flow control velocity. This also results in a non-linear correlation.

The stability analysis currently being conducted tests the case for α = 7.5° with *Re* = 176 at a given value of C_Q to determine the critical *Re*, comparable to Sheard *et al.* (2009) to determine the effectiveness of the flow control mechanism to alter the occurrence of the instability mode

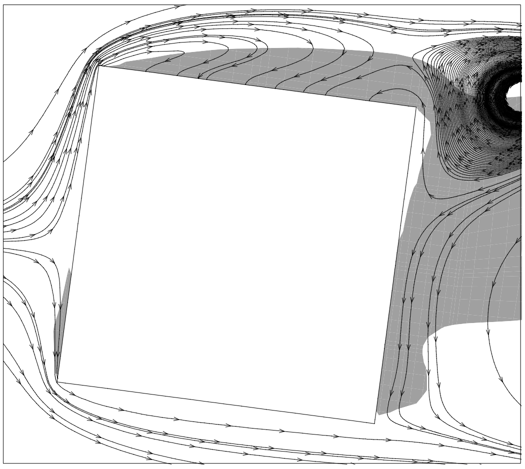

Figure 11: Streamline plot for $\alpha = 7.5°$, $Re = 176$ and $C_Q = -0.0333$ with no recirculation on upper surface

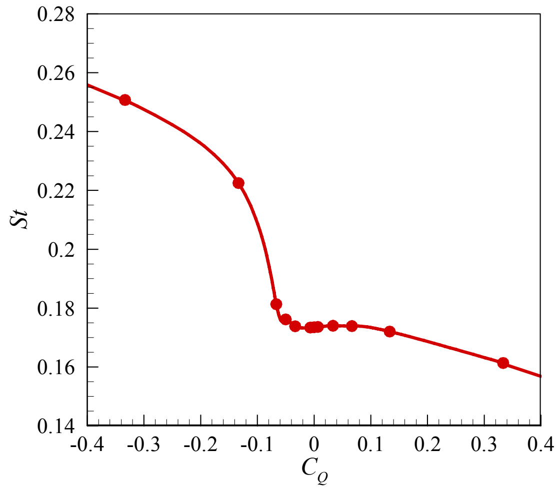

Figure 12: Strouhal number versus C_Q for $\alpha = 7.5°$, $Re = 176$. The solid line is provided for guidance.

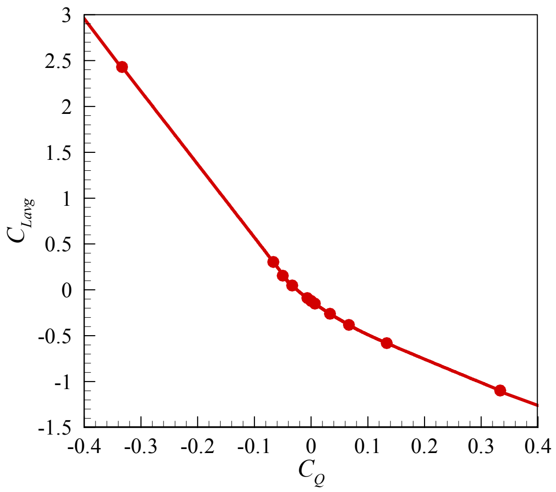

Figure 13: C_{Lavg} versus C_Q for $\alpha = 7.5°$, for $Re = 176$. The solid line is added to the data for guidance.

A, since $\alpha = 7.5°$ falls within the range dominated by mode A. A similar stability analysis will be required for the case with 15° angle of incidence. It is desired that the flow control mechanism will improve the critical *Re* significantly enough to warrant the use of this technique in engineering applications.

CONCLUSION

This paper presents the shear layer attachment on the upper surface of a square cylinder as a function of incidence angle for $\alpha = 0°$, 7.5° and 15°, in addition to imposing a flow control mechanism to induce reattachment. Trends observed for $\alpha = 7.5°$ and 15° are converse to those seen for $\alpha = 0°$ in terms of location, extent and duration of reattachment during a period of the wake shedding cycle. The ongoing stability analysis looks at the 7.5° case at a given value of C_Q to determine the influence of the flow control mechanism on the critical *Re* for transition from two-dimensional to three-dimensional flow. As a result, this will confirm any association between the instability modes A and C as outlined in Sheard *et al.* (2009) and the incidence angle. While the flow control mechanism eliminates any reattachment and separation at fairly low values of C_Q, the correlation between the average lift coefficient for the cylinder and the flow control coefficient is non-linear. Interestingly, the relationship between the wake shedding frequency and the flow control coefficient is also non-linear. This distinct, non-linear correlation is one of the fundamental concepts in understanding how to use this technique effectively.

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